Online Planning of Multi-Segment Trajectories with Trigonometric blends

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Abstract: In this paper, the approach based on FIR (Finite Impulse Response) filters, that has been shown to be very efficient for planning time-optimal trajectories composed by polynomial segments, is extended to the design of trajectories characterized by profiles of velocity, acceleration, jerk or even higher derivatives composed by trigonometric functions. A simple discrete-time filter, able to provide as output this kind of trajectories when a rough input signal composed by step functions is applied, is proposed, with two main consequences: 1) the generation of the trajectory results very efficient, even with high degree of continuity and the planning can be performed online; 2) the equivalence between the considered class of trajectories and linear filters allows an immediate frequency characterization of the motion law. In this way, it is possible to define the trajectories by considering constraints expressed in the frequency-domain besides the classical time-domain specifications, such as bounds on velocity, acceleration, and so on. Two examples illustrates the main features of the proposed trajectory planner, in particular with respect to the problems of multi-point trajectory generation and residual vibrations suppression by proper reference inputs application.

Keywords: Trajectory planning, Shaping filters, Digital filters.

1. INTRODUCTION

The growing need of planning trajectories online has led to the development of a number of filters able produce motion profiles with the desired degree of smoothness starting from rough reference signals, such as step functions, which set the desired final position. In Zanasi and Morselli (2003); Zheng et al. (2009), time-optimal trajectory planners with bounds on velocity, acceleration and jerk have been proposed. Basically, these systems are composed by a chain of integrators (whose output represents the desired trajectory) properly feedback controlled in order to track in the fastest way the reference input while remaining compliant with the given constraints. More recently, Biagiotti and Melchiorri (2011) show that time-optimal multi-segment polynomial trajectories with constraints on the first n derivatives are equivalent to the outputs of a chain of n moving average filters (see Sec. 2 for a brief overview) and, in general FIR, (Finite Impulse Response) filters (with rectangular impulse responses, as in case of moving average filters, but also sinusoidal impulse responses) are used for smoothing given motion profiles, see Nozawa et al. (1985); Kim et al. (1994); Jeon and Ha (2000).

In many works the adoption of trigonometric functions is proposed with the purpose of planning trajectories with smoother acceleration or jerk profiles that reduce residual vibrations when applied to resonant systems, see Li et al. (2007), but to the best of the authors' knowledge, besides experimental evidences, no analytical explanations of the advantages that they involve are present in the literature. In this paper, the generation of trajectories with trigonometric (and in particular sinusoidal) velocity, acceleration or jerk (or higher derivatives) profiles is related to the design of dynamic systems composed by moving average filters and a special type of FIR filter with a sinusoidal impulse response. Similarly to time-optimal polynomial trajectories, the characteristic parameters of each filter can be computed on the basis of the desired bounds on velocity, acceleration, jerk, and so on. Furthermore, the equivalence between dynamic filters and trajectories expressed by analytic functions provides an immediate characterization of the motion from a spectral point of view. As a consequence, it is also possible to select the filters parameters with the purpose of properly shaping the frequency spectrum of the trajectory.

2. TIME-OPTIMAL MULTI-SEGMENT TRAJECTORIES AND DYNAMIC FILTERS

In Biagiotti and Melchiorri (2011), it is shown that a multi-segment trajectory $q_n(t)$ of order n, compliant with the symmetric constraints

$$q^{(i)}_{min} = -q^{(i)}_{max}, \qquad i = 1, \ldots, n+1$$

can be obtained by filtering a step input with a cascade of n dynamic filters, each one characterized by the transfer function

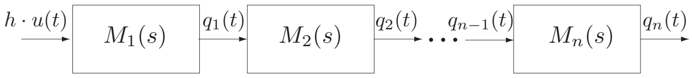


Fig. 1. System composed by n filters for the computation of an optimal trajectory of class $\mathcal{C}^{n-1}$.

$$M_i(s) = \frac{1}{T_i}\frac{1 - e^{-sT_i}}{s} \tag{1}$$

where the parameter T_i (in general different for each filter composing the chain) is a time length, see Fig. 1. In mathematical terms, this means that

$$q_n(t) = h \cdot u(t) * m_1(t) * m_2(t) * \ldots * m_n(t) \tag{2}$$

where $u(t)$ denotes the unit step function, h is the desired displacement, $m_i(t) = \mathcal{L}^{-1}\{M_i(s)\}$ is the impulse response of each filter, and $*$ indicates the continuous time convolution operator. The smoothness of the trajectory, that is the number of continuous derivatives, is strictly tied to the number of filters composing the chain. If we consider n filters, the resulting trajectory will be of class $\mathcal{C}^{n-1}$. By increasing the smoothness of the trajectory, its duration augments as well. As a matter of fact, the total duration of a trajectory planned by means of n dynamic systems $M_i(s)$ is given by the sum of the lengths of impulse response of each filter, i.e.

$$T_{tot} = T_1 + T_2 + \ldots + T_n.$$

The parameters T_i can be set with the purpose of imposing desired bounds on velocity, acceleration, jerk and higher derivatives, by assuming

$$T_1 = \frac{|h|}{q_{max}^{(1)}} \quad \text{and} \quad T_i = \frac{q_{max}^{(i-1)}}{q_{max}^{(i)}}, \quad i = 2, \ldots, n \tag{3}$$

with the constraints

$$T_i \geq T_n + \ldots + T_{i+1}, \qquad i = 1, \ldots, n-1.$$

These conditions are necessary to guarantee that the output trajectory never exceeds the limits $q_{max}^{(i)}$.

In order to evaluate the trajectory at discrete time instants kT_s, being T_s the sampling period, the system composed by n filters may be discretized by applying on each filter $M_i(s)$ the backward differences method that leads to the expression of a moving average filter

$$M_i(z) = \frac{1}{N_i}\frac{1 - z^{-N_i}}{1 - z^{-1}} \tag{4}$$

where $N_i = T_i/T_s$ denotes the number of samples (not null) of the filter response. Therefore, the implementation of the proposed trajectory generator on a digital controller can be achieved by considering the function $M_i(z)$ in lieu of the corresponding function $M_i(s)$ in the block-scheme of Fig. 1. Note that the digital implementation of each filter only requires two additions and one multiplication. As a consequence, even for high values of the degree n, the trajectory generator (composed by n filters) results very efficient from a computation point of view.

Finally, the structure of the trajectory planner, composed by linear filters, provides an immediate characterization of the resulting trajectories in terms of frequency content. By Fourier transforming (2), one obtains that the expression of the amplitude spectrum of $q_n(t)$ and of its derivatives $q_n^{(k)}(t)$ of generic order k results

$$|Q_n^{(k)}(j\omega)| = h \cdot |\omega^{k-1}| \cdot |M_1(j\omega)| \cdot |M_2(j\omega)| \cdot \ldots \cdot |M_n(j\omega)|$$

with

$$|M_i(j\omega)| = \left|\operatorname{sinc}\left(\frac{\omega}{\omega_i}\right)\right| \tag{5}$$

where $\operatorname{sinc}(\cdot)$ denotes the normalized sinc function defined as $\operatorname{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$ and $\omega_i = \frac{2\pi}{T_i}$. Note that the function $|M_i(j\omega)|$ is null for $\omega = k\,\omega_i$, with k integer. This property can be profitably exploited to properly choose the parameters of the trajectory/filter with the purpose of nullifying the spectrum of the trajectory at critical frequencies, for instance the eigenfrequencies of the plant. For this aim, if ω_r denotes a resonant frequency, it is sufficient to assume

$$\omega_i = \frac{\omega_r}{l} \quad \Leftrightarrow \quad T_i = l\,\frac{2\pi}{\omega_r}, \quad l = 1, 2, \ldots. \tag{6}$$

3. MULTI-SEGMENT TRAJECTORIES WITH POLYNOMIAL AND TRIGONOMETRIC SEGMENTS

The same structure of the filter for the generation of time-optimal trajectories, reported in Fig. 1, can be exploited to plan motion profiles characterized by velocity, acceleration, or jerk (or higher derivatives, depending on the order of the trajectory) composed only by sinusoidal functions, leading to the so-called modified trapezoidal velocity trajectory, modified double-S velocity trajectory, etc., see Biagiotti and Melchiorri (2008). In this case, it is sufficient to consider in the chain of averaging filters $M_i(s)$, characterized by a rectangular impulse response, a single filter whose impulse response is

$$s_i(t) = \begin{cases} \dfrac{\pi}{2T_i}\sin\left(\dfrac{\pi}{T_i}t\right) & \text{if } 0 \leq t \leq T_i \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

$$= \frac{\pi}{2T_i}\left[\sin\left(\frac{\pi}{T_i}t\right)u(t) + \sin\left(\frac{\pi}{T_i}(t - T_i)\right)u(t - T_i)\right]$$

where $u(t)$ denotes again the step function, and T_i is a parameter that defines the time duration of the response, which is finite as shown in Fig. 2. By Laplace transforming (7), the transfer function of the filter can be readily obtained:

$$S_i(s) = \frac{1}{2}\left(\frac{\pi}{T_i}\right)^2 \frac{1 + e^{-sT_i}}{s^2 + \left(\dfrac{\pi}{T_i}\right)^2}. \tag{8}$$

Note that the system $S_i(s)$ has a unitary dc gain.

The generation of a trajectory $q_{n,h}(t)$ whose n-th derivative is only composed by sinusoidal functions (and therefore is

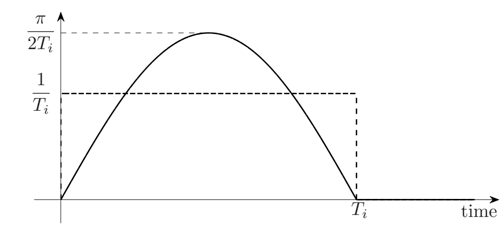


Fig. 2. Impulse response of the filter $S_i(s)$ defined by (8) (solid line) compared with that of an average filter $M_i(s)$ (dashed line) characterized by the same time constant T_i.

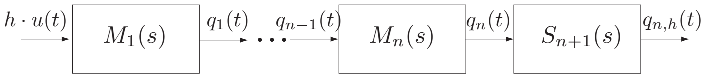


Fig. 3. System composed by $n+1$ filters for the computation of the trajectory $q_{n,h}(t)$ of class $\mathcal{C}^{n+1}$, whose n-th derivative is only composed by sinusoidal functions.

of class $\mathcal{C}^{n+1}$) can be achieved by adding the "sinusoidal" filter $S_{n+1}(s)$ at the end of a chain of n filters $M_i(s)$, as shown in Fig. 3. With this configuration, it is possible to find the following relation between the maximum values of $q^{(n)}(t)$ and $q^{(n+1)}(t)$ and the characteristic parameter T_{n+1} of the filter:

$$q^{(n)}_{max}(t)\frac{\pi}{2T_{n+1}} = q^{(n+1)}_{max}(t).$$

As a consequence, if constraints on the n-th and $(n+1)$-th derivative are given, the time-length T_{n+1} can be computed as

$$T_{n+1} = \frac{q^{(n)}_{max}}{q^{(n+1)}_{max}}\frac{\pi}{2}. \tag{9}$$

3.1 Spectral characterization of the trajectory

The spectral contents of trajectory $q_{n,h}(t)$, provided by the generator of Fig. 3 fed by input step functions, is given by the contribution of all the $n+1$ filters and by the input, i.e.

$$Q_{n,h}(j\omega) = \frac{h}{j\omega}\cdot M_1(j\omega)\cdot \ldots \cdot M_n(j\omega)\cdot S_{n+1}(j\omega)$$

where $S_i(j\omega)$, with $i = n+1$, is the frequency response of the sinusoidal filter:

$$S_i(j\omega) = \left(\frac{\pi}{T_i}\right)^2 e^{-j\frac{\omega T_i}{2}}\frac{\cos\left(\frac{\omega T_i}{2}\right)}{-\omega^2+\left(\frac{\pi}{T_i}\right)^2}.$$

In particular, the amplitude spectrum of the generic k-th derivative of the trajectory ($q^{(0)}_{n,h} = q_{n,h}$) is

$$|Q^{(k)}_{n,h}(j\omega)| = h\cdot|\omega^{k-1}|\cdot|M_1(j\omega)|\cdot\ldots\cdot|M_n(j\omega)|\cdot|S_{n+1}(j\omega)|$$

where $M_i(j\omega)$ are defined in (5) and

$$|S_i(j\omega)| = \left(\frac{\pi}{T_i}\right)^2\frac{\left|\cos\left(\frac{\omega T_i}{2}\right)\right|}{\left|-\omega^2+\left(\frac{\pi}{T_i}\right)^2\right|} = \frac{\left|\cos\left(\frac{\omega T_i}{2}\right)\right|}{\left|1-\left(\frac{\omega T_i}{\pi}\right)^2\right|} =$$

$$= \frac{\left|\cos\left(\pi\frac{\omega}{\omega_i}\right)\right|}{\left|1-\left(\frac{\omega}{\omega_i/2}\right)^2\right|}$$

with $\omega_i = \frac{2\pi}{T_i}$. In Fig. 4 the magnitude spectra $|S_i(j\omega)|$ and $|M_i(j\omega)|$ are compared. Note that $|S_i(j\omega)| = 0$ for $\omega = \frac{(2k+1)}{2}\frac{2\pi}{T_i}$, $k = 1, 2, \ldots$, and therefore the first zero ($\omega = \frac{3}{2}\omega_i$) is located at a frequency 1.5 times higher than the first zero of the filter $M_i(s)$ with the same time constant T_i. This means that, in order to nullify $|S_i(j\omega)|$ at a specific frequency ω_r, it is necessary to assume a time-length T_i which is 1.5 higher than the corresponding time period that assures $|M_i(j\omega_r)| = 0$, i.e.

$$\omega_i = \frac{2}{3}\frac{\omega_r}{l} \quad\Leftrightarrow\quad T_i = l\,\frac{3}{2}\frac{2\pi}{\omega_r}, \quad l = 1, 2, \ldots. \tag{10}$$

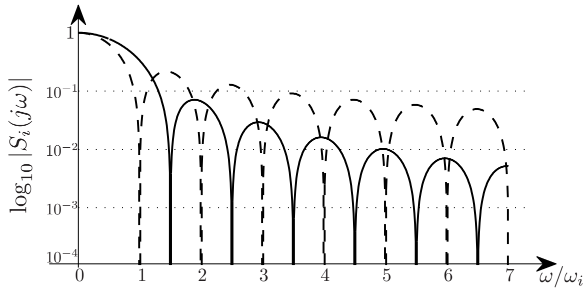


Fig. 4. Magnitude of the frequency response of $S_i(s)$ (solid line) compared with that of $M_i(s)$ (dashed line).

On the other hand, it is worth noticing that the decreasing of $|S_i(j\omega)|$ at high frequencies is quite faster than $|M_i(j\omega)|$, characterized by the same T_i. Therefore, by using a sinusoidal filter in lieu of an average filter the spectral contents of the output trajectory is considerably reduced at high frequencies. This explains the reason why trajectories with sine shaped velocity or acceleration, like harmonic trajectories and modified trapezoidal trajectories, lead to a consistent residual vibration reduction when applied to resonant systems, see Li et al. (2007). The equivalence between trajectories and dynamic filters allows to explain the effects of trigonometric functions in the definition of the trajectories and provide a systematic procedure to set their parameters with the purpose of properly shaping their spectral content.

3.2 Evaluation of the trajectory at discrete time instants

In order to evaluate the trajectory at time instants kT_s, it is necessary to discretize the filter of Fig. 3. The discrete transfer function $S_i(z)$ of the sinusoidal filter has been computed by z-transforming the sequence obtained by sampling (7) with a periods T_s:

$$S_i(z) = \frac{(1-\cos(\frac{\pi}{N_i}))(z^{-1}+z^{-(N_i+1)})}{1-2z^{-1}\cos(\frac{\pi}{N_i})+z^{-2}}$$

where $N_i = T_i/T_s$ In this way, the impulse response of the discrete-time filter coincides exactly with continuous one at discrete time instants kT_s, and is therefore zero for $kT_s > T_i$. Note that, being $\cos(\frac{\pi}{N_i})$ a constant to be computed only once, the digital implementation of $S_i(z)$ is computationally efficient, requiring four additions and two multiplications.

4. TRIGONOMETRIC TRAJECTORIES WITH MIN/MAX CONSTRAINTS

4.1 Trigonometric trajectory of order zero

The analytic expression of the trajectory $q_{0,h}(t)$, obtained by directly applying a step input function to the sinusoidal filter (8), can be deduced by integrating the impulse response of the filter:

$$q_{0,h}(t) = \frac{h}{2}\left(1-\cos\left(\frac{\pi}{T_1}t\right)\right)+q_0. \tag{11}$$

where q_0 is the starting point of the trajectory. Note that (11) is the standard expression of the harmonic trajectory, whose derivative is (7). In this case the unique

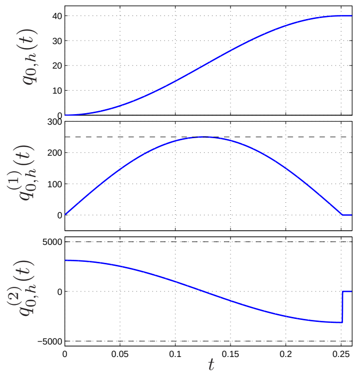
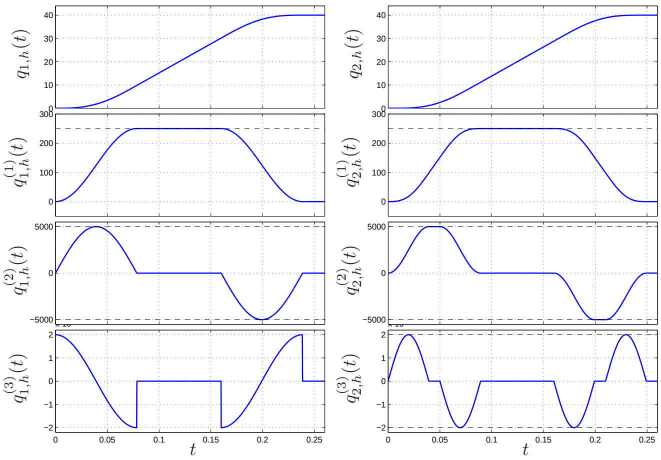


Fig. 5. Profiles of position, velocity and acceleration of the harmonic trajectory $q_{0,h}(t)$ obtained with $h = 40$rad and $\mathtt{v}_{max} = 250$rad/s ($T_1 = 0.2513$s).



(a) (b)

Fig. 6. Profiles of position, velocity and acceleration of the modified trapezoidal velocity trajectory $q_{1,h}$ obtained with $h = 40$rad and $\mathtt{v}_{max} = 250$rad/s and $\mathtt{a}_{max} = 5000$rad/s^2 ($T_1 = 0.16$s, $T_2 = 0.0785$s) (a) and of the modified double-S velocity trajectory $q_{2,h}$ obtained with the additional constraint $\mathtt{j}_{max} = 200000$rad/s^3 ($T_1 = 0.16$s, $T_2 = 0.05$s, $T_3 = 0.0393$s) (b).

free parameter T_1 can be set with the purpose of obtaining a prescribed maximum velocity, i.e.

$$T_1 = \frac{\pi}{2}\frac{h}{\mathtt{v}_{max}}$$

as shown in Fig. 5, or of locating the zeros of the frequency spectrum at precise critical frequencies.

4.2 Trigonometric trajectory of order $n \geq 1$

Trigonometric trajectory $q_{n,h}$ of order $n \geq 1$, can be easily obtained by adding average filters $M_i(s)$ into the chain of FIR filters. For instance, the trajectory $q_{1,h}$, corresponding to a modified trapezoidal velocity trajectory characterized by a sinusoidal acceleration, is generated by considering one filter $M_i(s)$ besides the sinusoidal filter (see Fig. 6(a)), while the use of two additional filters $M_i(s)$ leads to a modified double-S velocity trajectory $q_{2,h}$, whose jerk is composed by sine functions (see Fig. 6(b)). Note that the modified double-S velocity trajectory has been defined for the first time in this paper on the analogy of the modified trapezoidal velocity trajectory, since the complexity of its analytical expression makes its implementation and its use for practical application quite prohibitive. Conversely, the generation of this kind of trajectory, and more generally of trigonometric trajectories of any order n, with dynamic filters produces only a little increase of the computational burden.

The parameters T_i, $i = 1, \dots, n+1$ can be computed on the basis of the constraints on velocity, acceleration, jerk, etc., according to (3) and (9). Therefore if l constraints are given, the trajectory must be at least of order $n = l - 1$. For instance for $n = 1$

$$T_1 = \frac{h}{\mathtt{v}_{max}}, \qquad T_2 = \frac{\pi}{2}\frac{\mathtt{v}_{max}}{\mathtt{a}_{max}}$$

while for $n = 2$

$$T_1 = \frac{h}{\mathtt{v}_{max}}, \qquad T_2 = \frac{\mathtt{v}_{max}}{\mathtt{a}_{max}}, \qquad T_3 = \frac{\pi}{2}\frac{\mathtt{a}_{max}}{\mathtt{j}_{max}}$$

Note that the time constant T_{n+1} always corresponds to the sinusoidal filter.

5. CASE STUDIES

5.1 Multi-point trajectories

As already mentioned, the proposed filter can be used for online trajectory generation providing as input a staircase function, whose constant values are the desired via-points p_j, $j = 0, \dots, m$. In order to assure a constant maximum velocity $\mathtt{v}_{max}$, the first moving average filter must be characterized by a variable time constant

$$T_{1,j} = \frac{h_j}{\mathtt{v}_{max}}, \qquad \text{with } h_j = p_j - p_{j-1}, \quad j = 1, \dots, m$$

while, if the desired bounds are not changed, the other parameters T_i, $i = 2, \dots, n+1$ remain constant. This means that the order $N_{1,j} = T_{1,j}/T_s$ of the first FIR filter must be modified in runtime whenever the input function changes. For more details, see Biagiotti and Melchiorri (2011). In Fig. 7 a modified double-S trajectory passing trough a set of given points is shown.

5.2 Time- and frequency-domain specifications

In the previous example the parameters of the trajectory generator are obtained on the basis of constraints (velocity, acceleration, jerk) expressed in the time-domain. On the other hand, as already mentioned, it is also possible to take into account frequency constraints, that may arise because of critical frequencies of the plant that must track the motion profile. In order to show the advantage of the proposed procedure for trajectory planning (in particular with respect to standard multi-segment polynomial trajectories), we consider the motion system of Fig. 8, composed by two inertias with an elastic transmission lightly damped, see Lambrechts et al. (2005); Barre et al. (2005); Meckl and Arestides (1998), whose model (from the motor position q_m to the load position q_l) can be described by the following transfer function

$$G_{ml}(s) = \frac{Q_l(s)}{Q_m(s)} = \frac{2\delta\omega_n s + \omega_n^2}{s^2 + 2\delta\omega_n s + \omega_n^2} \qquad (12)$$

with

$$\omega_n = \sqrt{\frac{k_t}{J_l}}, \qquad \delta = \frac{b_t}{2\sqrt{k_t J_l}}.$$

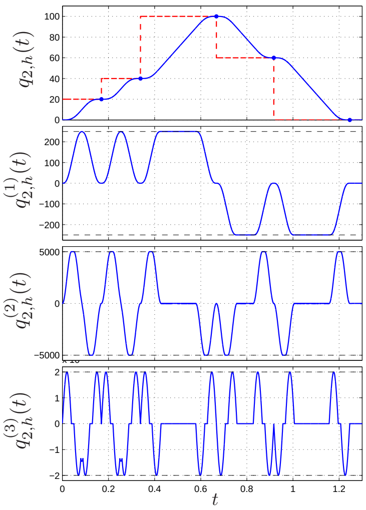


Fig. 7. Complex motion obtained with a second order trigonometric trajectory passing through a sequence of via-points $\boldsymbol{p} = \{0, 20, 40, 100, 60, 0\}$, with the same constraints of Fig. 6(b).

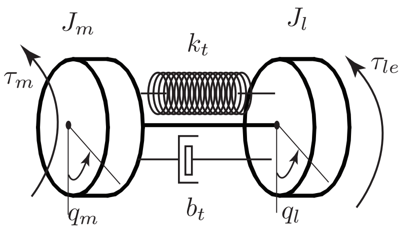


Fig. 8. Lumped constant model of a motion system with elastic transmission.

Parameter	Symbol	Value	Unit
Motor inertia	J_m	0.72×10^{-5}	$\mathrm{kg\,m^2}$
Load inertia	J_l	0.23×10^{-5}	$\mathrm{kg\,m^2}$
Spring stiffness	k_t	0.156	$\mathrm{N\,m}$
Internal damping	b_t	1.0×10^{-5}	$\mathrm{N\,m\,s}$

Table 1. Motion system parameters.

The parameters of the system, reported in Tab. 1, are derived from Lambrechts et al. (2005), as well as the trajectory constraints, that have been already adopted in the examples of Fig. 6(b) and Fig. 7. The resonant frequency of the system results $\omega_r = \omega_n \approx$ 260 rad/s.

We suppose that an ideal control system imposes to the (rotor) inertia J_m the desired motion profile, that is $q_m(t) = q_{ref}(t)$, being $q_{ref}(t)$ a trajectory obtained with the filter proposed in previous sections, and we analyze the effects of a particular choice of the trajectory parameters on the dynamic behavior of the plant and in particular on the tracking error, defined as $\varepsilon(t) = q_{ref}(t) - q_l(t) = q_m(t) - q_l(t)$. Obviously, the choice of the parameters of the filter is critical only if the spectral components of the trajectory are appreciable in the neighborhood of the eigenfrequency of the plant.

In order to meet the time-domain limits $\mathtt{v}_{max}$, $\mathtt{a}_{max}$ and the additional constraints $|Q_{n,h}(j\omega_r)| = 0$, the order of the multi-segment trigonometric trajectory must be $n = 2$ (modified double-S velocity trajectory). In this way, the parameters T_i are set as

$$T_1 = \frac{h}{\mathtt{v}_{max}} = 0.16\text{s},\ T_2 = \frac{\mathtt{v}_{max}}{\mathtt{a}_{max}} = 0.05\text{s},\ T_3 = \frac{3}{2}\frac{2\pi}{\omega_3} = 0.0362\text{s}.$$

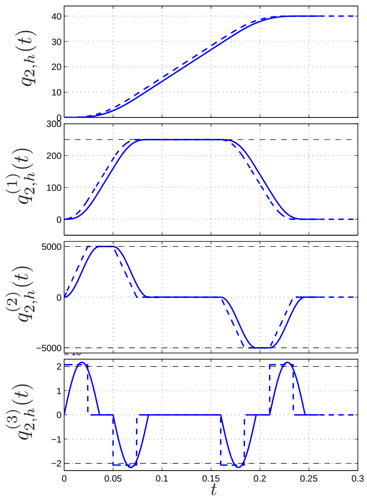
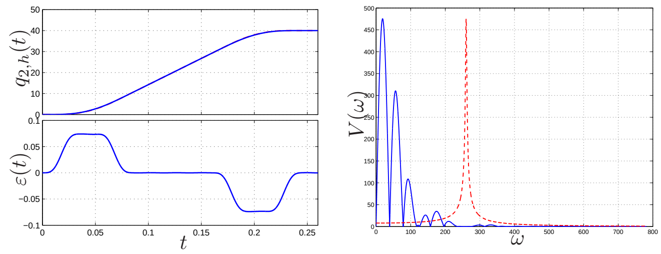


Fig. 9. Profiles of position, velocity and acceleration of a modified double-S velocity trajectory (solid line) and of a double-S trajectory (dashed line) computed under the same conditions.



(a) (b)

Fig. 10. Response of the elastic system G_{ml} to a modified double-S velocity trajectory: tracking error (a) and frequency spectrum of the acceleration (b).

In Fig. 9 the profiles of position, velocity, acceleration and jerk of the trigonometric trajectory are compared with those of a standard double-S velocity trajectory, defined by

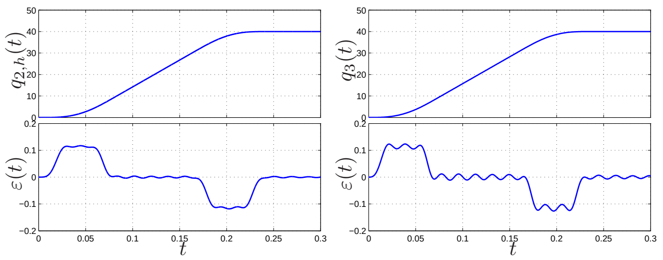
$$T_1 = \frac{h}{\mathtt{v}_{max}} = 0.16\text{s},\ T_2 = \frac{\mathtt{v}_{max}}{\mathtt{a}_{max}} = 0.05\text{s},\ T_3 = \frac{2\pi}{\omega_r} = 0.0209\text{s}.$$

In the nominal case both trajectories behave pretty well, since they do not cause residual vibrations at the end of motion. For instance, in Fig. 10(a) the tracking error obtained with a modified double-S velocity trajectory is shown. At the end of motion the error is null and does not present oscillation. This is due to the fact that at ω_r the frequency spectrum of the trajectory is null. In particular, in Fig. 10(b) the spectrum $|Q_{n,h}^{(2)}(j\omega)|$ of the acceleration profile, hereafter denoted with $V(\omega)$, is compared with the magnitude of the frequency response of

$$G_\varepsilon(s) = \frac{1}{s^2 + 2\delta\omega_n s + \omega_n^2}.$$

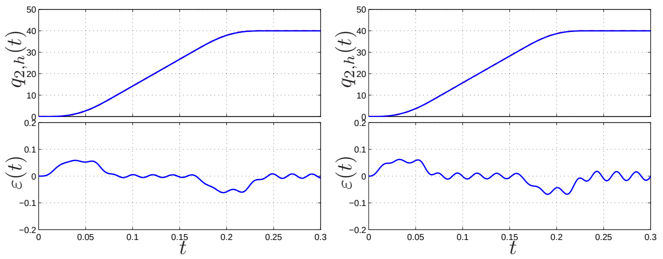
since $G_\varepsilon(s)$ defines the dynamic relation between the acceleration profile $q_{n,h}^{(2)}(t)$ and the tracking error $\varepsilon(t)$. Note that in the plot $|G_\varepsilon(j\omega)|$ has been properly scaled for the sake of clarity.

The advantages of modified trigonometric trajectories over standard multi-segment polynomial trajectories come out when the real elastic system differs from the nominal model $G_{ml}(s)$ used to design the trajectory. As a matter of fact, trigonometric trajectories are more robust when



(a) (b)

Fig. 11. Response of an elastic system with two vibrational modes to a modified double-S velocity trajectory (a) and to a double S velocity trajectory (b) computed under the same conditions.



(a) (b)

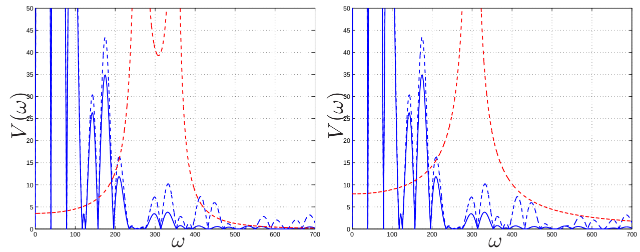
Fig. 12. Response of an elastic system with a resonant frequency different from the nominal one (used to define the input trajectories) to a modified double-S velocity trajectory (a) and to a double S velocity trajectory (b) computed under the same conditions.

the system has unmodeled resonant modes, as in the case of Fig. 11 where an additional resonant frequency at $\omega_{r,2} = 350$rad/s has been considered, or when the real eigenfrequency differs from the nominal one, as in the example reported in Fig. 12 where $\omega_r = 300$rad/s.

In the case of a second unmodeled resonant mode the magnitude of the residual vibrations (that is the peak to peak value of the tracking error $\varepsilon(t)$ for $t > T_{tot}$) is $\varepsilon_{p2p} = 0.0051$rad for the modified double-S trajectory, while for the double-S trajectory is $\varepsilon_{p2p} = 0.0140$rad. The wrong estimation of the resonant frequency leads to $\varepsilon_{p2p} = 0.0164$rad for the modified double-S trajectory and $\varepsilon_{p2p} = 0.0363$rad for the double-S trajectory. A comparative analysis of the frequency spectra of the two different trajectories, reported in Fig. 13, reveals the reason of this result: for frequencies higher than ω_r the magnitude of spectral components of $q_{2,h}(t)$ is considerably lower than the magnitude of the components of $q_3(t)$.

6. CONCLUSIONS

In this paper, the generation of trajectories with trigonometric velocity, acceleration or jerk (or higher derivatives) profiles is achieved by means of dynamic systems composed by moving average filters and a special type of FIR filter with a sinusoidal impulse response. Similarly to time-optimal polynomial trajectories, the characteristic parameters of each filter can be computed on the basis of the desired bounds on velocity, acceleration, jerk, and so on. Furthermore, the equivalence between dynamic filters and trajectories expressed by analytic functions provides an immediate characterization of the motion from a spectral point of view. As a consequences, it is also possible to select the filters parameters with the purpose of properly shaping the frequency spectrum of the resulting trajectory.



(a) (b)

Fig. 13. Frequency spectrum of a modified double-S velocity trajectory (solid line) and of a double S velocity trajectory (dashed line) compared with the frequency response of an elastic system with two vibrational modes (a) and with a resonant frequency different from the nominal one (b).

With this respect, the use of trigonometric trajectories leads to more robust results than multi-segment polynomial trajectories. As a matter of fact when they are applied to resonant systems with uncertain parameters or unmodeled dynamics, modified trigonometric trajectories allow a noticeable reduction of residual vibrations, even if the resonant frequency is not correctly estimated.

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